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Euler's Homogenous Functions Can Describe Non-extensive Thermodynamic Systems

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Abstract In this work by using the new concepts of nanothermodynamics as well as the exact definitions of intensive and extensive thermodynamic properties a new mathematical interpretation of these properties has been proposed. It is demonstrated that Euler's homogeneous functions with a non-integer order can define the thermodynamic functions of the nonextensive systems.

Keywords: Euler's functions, Nonextensive thermodynamic systems, Entropy.

1. Introduction

Non-extensive thermodynamics [1] is a new branch of thermodynamics that has a close relation to nanothermodynamics [2-6]. It is used to study of those physical systems that have not the property of extensivity. Rigorous definitions have been made to present this property from thermodynamic point of view [7,8]. Thanks to Tsallis [1,9] for his pioneering work on the new entropy definition, which could be written as:

$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad ; \quad (\sum_{i=1}^{W} p_i = 1, q \in \mathbb{R}) \tag{1.1}$$

where q is entropic index parameter, k is a positive constant (Boltzmann's constant), and W is the total number of microscopic possibilities of the system. Depending to values of this parameter different cases exist. For $q > 1$ we have subadditivity (subextensivity), $q = 1$ goes back to additivity (extensivity) and $q < 1$ represents superadditivity (superextensivity) [1]. In case of $q = 1$ ordinary thermodynamic relations will be recovered which is the basic formula of Boltzmann for defining entropy:

$$S = -k\sum_{i=1}^{W} p_i ln p_i \tag{1.2}$$

2. The Euler's Homogeneous Functions and Their Applications to Thermodynamic Properties

Consider a function $f(a, b, x, y)$ which is homogeneous to the degree $h > 0$ in x and y and to degree 0 in a and b. By definition, if the variables x and y are each multiplied by a factor k, the value of $f(a, b, kx, ky)$ will be increased by a factor k^h. Thus, for any value of k, we have

$$f(a, b, X, Y) = k^h f(a, b, x, y) \quad ; \quad (X = kxandY = ky) \tag{2.1}$$

In conventional thermodynamics there are some functions of interest of us that are special cases of homogeneous functions. In particular, these functions are either homogeneous to the first degree in mass (extensive) with $h = 1.0$ or homogeneous to the zeroth degree in mass (intensive) with $h = 0$. The arbitrary multiplier, k, will always be equal to the mass or moles of the system, N (or $1/N$ as the case may be).

3. Concluding Remarks

There are experimental observations [10] that show various thermodynamic properties such as entropy have dependence to the number of atoms. This important point shows that entropy of a small number of atoms is no longer an extensive thermodynamic property or from mathematical point of view this property is not a first order Euler homogeneous function. It means that h in equation (2.1) differs from unity. In this work we propose a real number for h. Further experimental data would be needed to determine the exact numerical value of this parameter as well as its probable universality.

References

[1] C. Tsallis, Nonextensive statistics: theoretical, experimental and computational evidences and connections, Brazilian Journal of Physics, **29**(1999), No. 1, 1-35.

[2] T.L. Hill, A different approach to nanothermodynamics, Nano Letters, **1**(2001), No. 5, 273-275.

[3] T.L. Hill, Thermodynamics of Small Systems, Parts 1 and 2, (Benjamin Co.) 1964.

[4] T.L. Hill and R. V. Chamberlin, Fluctuations in energy in completely open small systems, Nano Letters, **2**(2002), No. 6, 609-613.

[5] A.K. Rajagopal, C. S. Pande, S. Abe, Nanothermodynamics-A generic approach to material properties at nanoscale, Invited presentation at the Indo-US Workshop on "Nanoscale Materials: From Science to Technology", Puri, India, April 5-8, 2004.

[6] G.R. Vakili-Nezhaad and G.A. Mansoori, An application of nonextensive statistical mechanics to nanosystems, to appear in the Journal of Theoretical and Computational Nanoscience, 2004.

[7] D.H.E. Gross, Microcanonical Thermodynamics, World Scientific, 2001.

[8] J.S. Tester and M. Model, Thermodynamics and Its Applications, 3rd Ed., Prentice -Hall, Inc., 1997.

[9] C. Tsallis, Possible generalization of Boltzmann-Gibbs statistics, J. Stat. Phys., **52**(1988), 479-487.

[10] D.H.E. Gross, A new thermodynamics from nuclei to stars, Entropy, **6** (2004), 158-179.