International Journal of Applied Mathematical Sciences.
ISSN 0973-0176 Vol.3 No.2 (2006), pp. 135-140
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http://www.gbspublisher.com/ijams.htm

Interpolation and Forecasting of Time Series

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Abstract

We discuss and compare several traditional methods of forecasting and interpolation methods of forecasting.

Mathematics Subject Classification(2000):Primary 42C40, 42A99, 42C99, 65D99.
Keywords: cubic and symmetric extentions, scaling function interpolation.

1. Introduction

The histrory and usefulness of interpolation have received attentions increasingly from many scientists and engineerings [4]. Many methods of interpolation or extrapolation have been used to perform data analysis or prediction. Understanding the time series or analyzing the data is essential for further applications such as forecasting. Traditional methods, such as Fourier methods, spline methods, are basically local extentions of given data. In this paper we illustrate some of these methods and introduce some interpolation methods which are more global in nature. The outline of this paper is as follows. In the second section, we present several traditional methods of forecasting by using cubic and symmetric extensions. We then explain how to use interpolation method for forecasting in section 3 followed by concluding remarks in section 4.

2. Cubic and Symmetric Methods

We use an example of signal extension to illustrate several traditional methods of forecasting. Considering a given original signal (figure 1), we extend it by using several different methods; cubic extension and symmetric extension (figure 2-4). It turns out that the extensions become monotonic extensions as they are extended. This means that

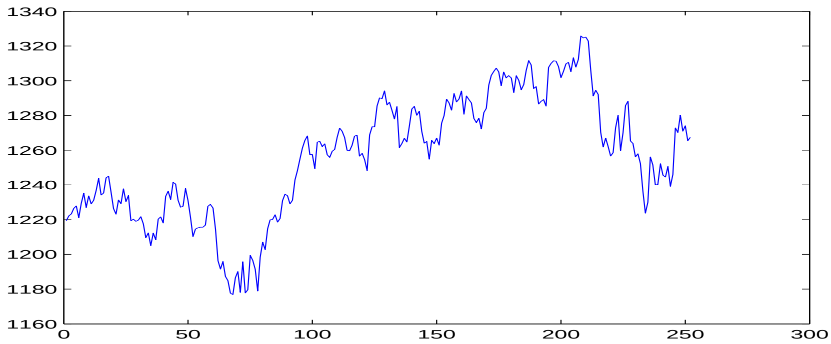

Figure 1: *Original signal*

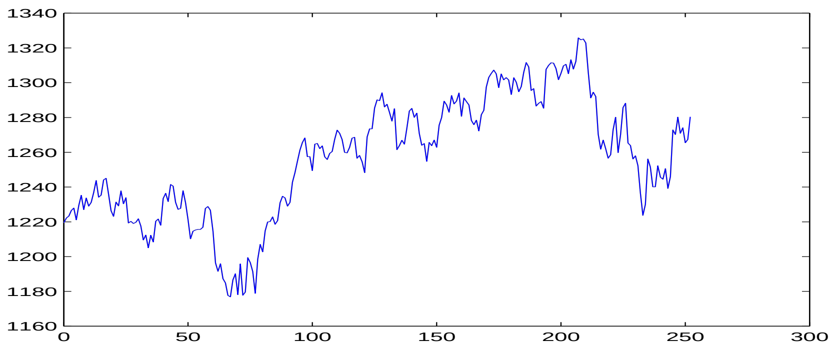

Figure 2: *Cubic extension of original signal by one unit*

these methods are more local in nature. One can only predict the extension in a small period of time rather than a longer period of time. To overcome this shortcoming, we propose interpolation method in the following section which explains a more global nature can be obtained.

3. Interpolation Methods

We first present some interpolation and approximation methods. Then we apply to forecasting. Multiresolution based interpolation methods are relatively new in approximation theory and become useful and powerful in several areas of mathematical analysis and applications. To review several approximation results obtained by using scaling function interpolation, we first recall the setting of multiresolution analysis.

Suppose $\phi(x)$ and $\psi(x)$ are the scaling function and the corresponding wavelet respectively with finite support [0, *l*] where l is a positive number. It is well known that $\phi(x)$ and $\psi(x)$ satisfy the following dilation equation and wavelet equation, respectively:

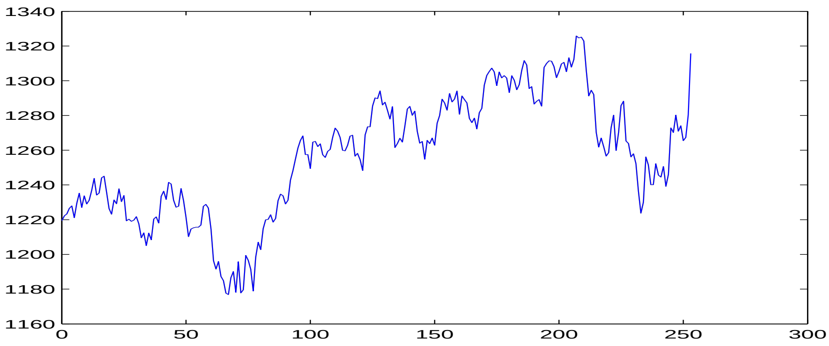

Figure 3: *Cubic extension of original signal by two units*

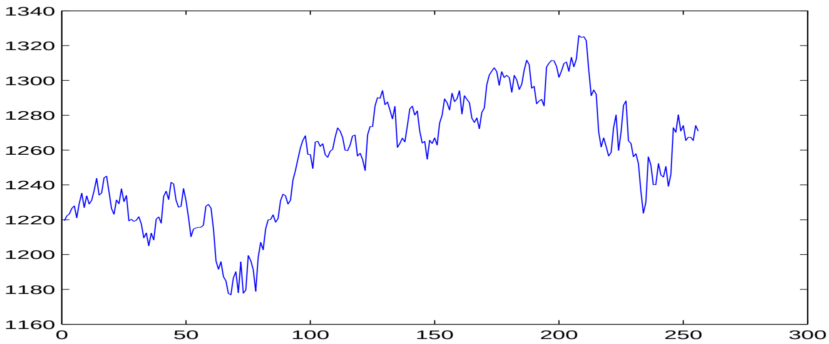

Figure 4: *Symmetric extension of original signal*

$$\phi(x) = \sqrt{2} \sum_{s=0}^{l} h_s \phi(2x - s)$$

and

$$\psi(x) = \sqrt{2} \sum_{s=0}^{l} g_s \phi(2x - s) \ ,$$

where the $h_s's$ and $g_s's$ are constants called low pass and high pass filter coefficients respectively.

We will use the following standard notations:

$$\phi_k^j(x) = 2^{\frac{j}{2}} \phi(2^j x - k),$$

and

$$\psi_k^j(x) = 2^{\frac{j}{2}} \phi(2^j x - k).$$

Consider the subspace V_j of L^2 defined by:

$$V_j = Span\{\phi_k^j, k \in \mathbb{Z}\},$$

and the subspace W_j of L^2 defined by :

$$W_j = Span\{\psi_k^j, k \in \mathbb{Z}\},$$

the subspaces $V_j's, -\infty < j < \infty$, form a multiresolution of L^2 with the subspace W_j being the difference between V_j and V_{j-1}. In fact, the L^2 space has an orthonormal decomposition as :

$$L^2 = V_j \bigoplus \sum_{J=j}^{\infty} W_J.$$

The projection of a L^2 function $f(x)$ onto the subspace V_j is defined by:

$$f_j(x) = \sum_k \alpha_{j,k} \phi_k^j(x),$$

where

$$\alpha_{j,k} = \int f(x) \phi_k^j(x) dx.$$

Similarly, we can project $f(x)$ onto W_j by:

$$w_j(x) = \sum_k \beta_{j,k} \psi_k^j(x),$$

where

$$\beta_{j,k} = \int f(x) \psi_k^j(x) dx.$$

Therefore, the function $f(x)$ can be decomposed by:

$$f(x) = f_j(x) + \sum_{i=j}^{\infty} w_i(x).$$

The projection $f_j(x)$ is called the *linear approximation* of the function $f(x)$ in the subspace V_j [7]. The multiresolution analysis of $L^2(R^2)$ can be defined in a similar fashion.

In [3], E. B. Lin and X. Zhou gave the following wavelet approximation theorem. In what follows, we assume ϕ, ψ are sufficiently smooth and satisfy the orthonormal multiresolution analysis with compact support and ψ has N vanishing moments. Let $\{M_l\}$ denote the moments of the scaling function ϕ, i.e.

$$M_l := \int x^l \phi(x) dx, \quad l = 1, 2, \cdots .$$

In particular,

$$M_1 = c := \frac{1}{2}\sum_k k a_k,$$

where $a_k = \sqrt{2}h_k$ are coefficients of the dilation equation.

Theorem 3.1. *Assume the function* $f \in C^k(\bar{\Omega})$*, where* Ω *is a bounded open set in* $R^2, k \geq N \geq 2$*. Let, for* $j \in Z$*,*

$$f^j(x,y) := \frac{1}{2^j}\sum_{p,q\in\wedge} f(\frac{p+c}{2^j}, \frac{q+c}{2^j})\phi_p^j(x)\phi_q^j(y), \ \ (x,y) \in \Omega, \tag{3.1}$$

where the index set $\wedge = \{(p,q)|(supp(\phi_p^j) \otimes supp(\phi_q^j)) \cap \Omega \neq \phi\}$*,*

In addition the moments M_i *satisfy*

$$M_i = c^i, \ \ i = 1, 2, \cdots, N-1.$$

i.e.

$$\int x^i\phi(x)dx = (\int x\phi(x)dx)^i, i = 1, 2, \cdots, N-1.$$

Then

$$||f - f^j||_{L^2(\Omega)} \leq C||f^{(N)}||_\infty 2^{-jN},$$
$$||f - f^j||_{H^1(\Omega)} \leq C||f^{(N)}||_\infty 2^{-j(N-1)},$$

where C is a constant depending only on N, diameter of Ω *and*

$$||f^{(N)}||_\infty := max_{(x,y)\in\Omega, m=0,1,\cdots,N} \, |\frac{\partial^N f}{\partial x^m \partial y^{N-m}}(x,y)|.$$

A new scheme of interpolation of a multivariate function with the tensor products of compactly supported multi-scaling functions was introduced in [2]. The idea is to choose weighted sample values of a given function by suitable weights. The advantages of multiwavelets versus scalar wavelets are possessing symmetry, orthogonality, short support and high approximation order [1],[5],[6]. A similar result to Theorem 3.1 in multi-scaling case was obtained in [2], namely, given a multivariate function $f \in C_0^N$, the interpolation scheme provides, at scale 2^j, an approximation order N in the L_2 space or an approximation order N-n in the Sobelev space H^n if and only if the multi-scaling function has the accuracy N.

In the context of the multi-scaling functions, one usually deals with a matrix dilation (refinement) equation:

$$\Phi(t) = \sum_k C_k \Phi(2t-k). \tag{3.2}$$

Here $\Phi(t) = [\Phi_0(t), \cdots, \Phi_{r-1}(t)]^T \in L^2$ is a multi-scaling function and C_k are r by r matrices with constant elements. If $r = 1, \ C_k$ are real numbers and $\Phi(t) = \Phi_0(t)$

which is called scalar scaling function. A characterization of the multi-scaling function can be found in [2].

One dimensional version of the interpolation formula can also be obtained and can be applied to extend the original signal.

4. Remarks

The above interpolation methods provide more global extension than traditional methods. The cubic or symmetric forecasting method bases on several points to extend to few more points, on the other hand, the scaling function interpolation methods can extend the given data to a longer period of interval. We plan to produce some examples and provide error comparisons by applying the scaling function interpolation.

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