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Numerical Method for Solving Obstacle Scattering Problems by an Algorithm Based on the Modified Rayleigh Conjecture

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Abstract

A numerical algorithm for solving obstacle scattering problems is presented. This algorithm is based on the Modified Rayleigh Conjecture (MRC) method introduced by A. G. Ramm. Its numerical implementation shows that the method is efficient, can handle nonsmooth 3D obstacles and obstacles with complicated shapes. Numerical examples are given.

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1. Introduction

The classical Rayleigh Conjecture is discussed in [4]-[6], where it is shown that, in general, this conjecture is incorrect: there are obstacles (for example, sufficiently elongated ellipsoids) for which the series, representing the scattered field outside a ball containing the obstacle, does not converge up to the boundary of this obstacle.

The Modified Rayleigh Conjecture (MRC) has been formulated and proved in [1],[15] (see Theorem 1 below). A numerical method for solving obstacle scattering problems, based on MRC, was proposed in [1]. This method was implemented in [2] for two-dimensional obstacle scattering problems. The numerical results in [2] were quite encouraging: they show that the method is efficient, economical, and is quite competitive compared with the usual boundary integral equations method (BIEM). A recent paper [3] contains a numerical implementation of MRC in some three-dimensional obstacle scattering problems. Its results reconfirm the practical efficiency of the MRC method. See also [11]-[14], where further developments and applications of the MRC method are given.

In this paper a numerical implementation of the Modified Rayleigh Conjecture (MRC) method for solving obstacle scattering problem in three-dimensional case is presented. Our aim is to consider more general than in [3] three-dimensional obstacles: non-convex, non-starshaped, non-smooth, and to study the performance of the MRC in these cases. The minimization problem (9) (see below), which is at the heart of the MRC method, is treated numerically in a new way, different from the one used in [2] and [3]. Our results present further numerical evidence of the practical efficiency of the MRC method for solving obstacle scattering problems.

The obstacle scattering problems (1)-(3), we are interested in, consists of solving the equation

$$(\bigtriangledown^2 + k^2)u = 0 \quad in \ \ D' = R^3 \setminus D, \tag{1}$$

where $D \subset R^3$ is a bounded domain, satisfies the Dirichlet boundary condition

$$u|_S = 0, \tag{2}$$

where S is the boundary of D, which is assumed Lipschitz in this paper, and the radiation condition at infinity:

$$u = u_0 + v = u_0 + A(\alpha', \alpha)\frac{e^{ikr}}{r} + o(\frac{1}{r}) \qquad r \to \infty, \tag{3}$$

$$r := |x|, \quad \alpha' = x/r, \quad u_0 := e^{ik\alpha\cdot x},$$

where v is the scattered field, $\alpha \in S^2$ is given, S^2 is the unit sphere in R^3, $k = const > 0$ is fixed, k is the wave number. The coefficient $A(\alpha', \alpha)$ is called the scattering amplitude.

Denote

$$A_l(\alpha) := \int_{S^2} A(\alpha', \alpha)\overline{Y_l(\alpha')}d\alpha', \tag{4}$$

where $Y_l(\alpha)$ are the orthonormal spherical harmonics:

$$Y_l = Y_{lm}, \quad -l \le m \le l, \quad l = 0, 1, 2, ...$$

$$Y_{lm}(\theta, \phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi}\Theta_{lm}(cos\theta),$$

$$\Theta_{lm}(x) = \sqrt{\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}}P_l^m(x),$$

$P_l^m(x)$ are the associated Legendre functions of the first kind,

$$P_l^m(x) := (1-x^2)^{m/2}\frac{d^m P_l(x)}{dx^m}, \quad m \geq 0,$$

and

$$P_l(x) := \frac{(-1)^l}{2^l l!}\frac{d^l}{dx^l}(1-x^2)^l.$$

For $m < 0$

$$\Theta_{lm}(x) = (-1)^m \Theta_{l,-m}(x).$$

Let $h_l(r)$ be the spherical Hankel functions of the first kind, normalized so that $h_l(kr) \sim e^{ikr}/r$ as $r \to +\infty$. Let $B_R := \{x : |x| \leq R\} \supset D$, and the origin is inside D.

Then in the region $r > R$, the solution to the acoustic wave problem (1)-(3) is of the form:

$$u(x,\alpha) = e^{ik\alpha\cdot x} + \sum_{l=0}^{\infty} A_l(\alpha)\psi_l(x), \ |x| > R,$$

$$\psi_l := Y_l(\alpha')h_l(kr), \qquad r > R, \quad \alpha' = x/r,$$

where

$$\sum_{l=0}^{\infty} := \sum_{l=0}^{\infty}\sum_{m=-l}^{l}.$$

Fix $\epsilon > 0$, an arbitrary small number. The following Lemmas and Theorem 1 are proved in [1],[15].

Lemma 1. *There exist $L = L(\epsilon)$ and numbers $c_l = c_l(\epsilon)$ such that*

$$||u_0(s) + \sum_{l=0}^{L} c_l(\epsilon)\psi_l(s)||_{L^2(S)} < \epsilon. \tag{5}$$

Lemma 2. *If (5) holds, then $||v_\epsilon(x) - v(x)|| = O(\epsilon), \ \forall x \in D', \quad \epsilon \to 0.$ where*

$$v_\epsilon(x) := \sum_{l=0}^{L} c_l(\epsilon)\psi_l(x), \qquad x \in D', \tag{6}$$

and

$$||.|| := ||.||_{H^m_{loc}(D')} + ||.||_{L^2(D';(1+|x|)^{-\gamma})}, \ \gamma > 0, m > 0, \tag{7}$$

m is arbitrary, and H^m is the Sobolev space.

Lemma 3. $c_l(\epsilon) \to A_l(\alpha), \forall l, \epsilon \to 0.$

Theorem 1 (Modified Rayleigh Conjecture). *Let $D \in R^3$ be a bounded obstacle with Lipschitz boundary S. For any $\epsilon > 0$ there exists $L = L(\epsilon)$ and $c_l(\epsilon) = c_{lm}(\epsilon)$, $0 \leq l \leq L$, $-l \leq m \leq l$, such that inequality (5) holds. If (5) holds then function (6) satisfies the estimate $||v(x) - v_\epsilon(x)|| = O(\epsilon)$, where the* *norm is defined in (7). Thus,* $v_\epsilon(x)$ *is an approximation of the scttered field everywhere in* D'.

In order to obtain an accurate solution, one usually takes L large. But as L grows the condition number of the matrix $(\psi_l,\ \psi_{l'})_{L^2(S)}$ is increasing very fast. So we choose some interior points $x_j \in D,\ j = 1, 2, ..., J$, and use the following version of Theorem 1 (see [2],[15]):

Theorem 2. *Suppose* $x_j \in D,\ j = 1, 2, ..., J$, *then* $\forall \epsilon > 0,\ \exists L = L(\epsilon)$ *and* $c_{lj}(\epsilon), l = 0, ..., L,\ j = 0, ..., J(\epsilon)$, *such that*

(i)

$$||u_0(s) + \sum_{j=0}^{J}\sum_{l=0}^{L} c_{lj}(\epsilon)\psi_l(s - x_j)||_{L^2(S)} < \epsilon. \tag{5'}$$

(ii)

$$||v_\epsilon(x) - v(x)|| = O(\epsilon),$$

where

$$v_\epsilon(x) = \sum_{j=0}^{J}\sum_{l=0}^{L} c_{lj}(\epsilon)\psi_l(s - x_j)$$

and the $||.||$ *is defined in Lemma 2.*

Remark. Theorem 1 is the basis for MRC algorithm for computation of the field scattered by an obstacle: one takes an $\epsilon > 0$ and an integer $L > 0$, minimizes the left-hand side of (5) with respect to c_l, and if the minimum is $\le \epsilon$ then the function (6) is the approximate solution of the obstacle scattering problem with the accuracy $O(\epsilon)$. If the above minimum is greater than ϵ, then one increases L until the minimum is less than ϵ. This is possible by Lemma 1. In computational practice, one may increase also the number J of points x_j inside D, as explained in Theorem 2. The increase of J allows one to reach the desired value of the above minimum keeping L relatively small. This gives computational advantage in many cases.

In section 2, an algorithm is presented for solving the problem (1)-(3). This algorithm is based on the MRC. Compared with the previous work in the case of two- and three-dimensional MRC([2],[3]), we consider more general surfaces, in particular non-starshaped and piecewise-smooth boundaries. The numerical results are given in section 3. A discussion of the numerical results is given in section 4.

2. The MRC algorithm for Solving Obstacle Scattering Problems

1. Smooth star-shaped boundary:

Assume the surface S is given by the equation

$$r = r(\theta, \varphi), \qquad 0 \le \varphi \le 2\pi,\ 0 \le \theta \le \pi.$$

Define

$$F(c_0, c_1, ..., c_L) := ||u_0 + \sum_{l=0}^{L} c_l \psi_l||^2_{L^2(S)}. \qquad (5'')$$

Let

$$h_1 = 2\pi/n_1, \qquad h_2 = \pi/n_2$$

$$0 = \varphi_0 < \varphi_1 < .... < \varphi_{n_1} = 2\pi,\ \varphi_{i_1} = i_1 h_1, \quad i_1 = 1, ..., n_1,$$

$$0 = \theta_0 < \theta_1 < .... < \theta_{n_1} = \pi,\ \theta_{i_2} = i_2 h_2, \quad i_2 = 1, ..., n_2,$$

where n_1 and n_2 are the number of steps. By Simpson's formula([8]), we obtain an approximation of $F(c_0, c_1, ..., c_L)$:

$$F(c_0, c_1, ..., c_L) = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2} + \sum_{l=0}^{L} c_l \psi_{l i_1 i_2}|^2 w_{i_1 i_2} h_1 h_2 \qquad (5''')$$

where

$$a_{i_1,i_2} = \begin{cases} 4, & i_1 \ and \ i_2 \ even \\ 8, & i_1 \ - \ i_2 \ odd \\ 16, & i_1 \ and \ i_2 \ odd \end{cases}$$

and

$$\psi_{l i_1 i_2} = Y_l(\theta_{i_1}, \varphi_{i_2}) h_l(kr(\theta_{i_1}, \varphi_{i_2})), \quad w_{i_1 i_2} = w(\theta_{i_1}, \varphi_{i_2})$$

where

$$w(\theta, \varphi) = (r^2 r_\varphi^2 + r^2 r_\theta^2 sin^2\theta + r^4 sin^2\theta)^{1/2}. \qquad (8)$$

There exists $c^* = (c_0^*, c_1^*, ..., c_L^*)$ such that

$$F(c^*) = min_{c_0,.....,c_L} F(c_0, c_1, ..., c_L). \qquad (9)$$

Write

$$F(c) = ||Ac - B||^2, \qquad (10)$$

where

$$A = (A_{l,i})_{M\times L_1}, \quad A_{l,i} = \psi_{l i_1 i_2} (a_{i_1 i_2} w_{i_1 i_2} h_1 h_2)^{\frac{1}{2}}, \quad i = i_1 i_2,$$

$$B = (B_i)_{M\times 1}, \quad B_i = u_{0 i_1 i_2} (a_{i_1 i_2} w_{i_1 i_2} h_1 h_2)^{\frac{1}{2}},$$

and $M = n_1 n_2$, $L_1 = (L+1)(2L+1)$ since $c_l = c_{lm}, 0 \le l \le L, -l \le m \le l$.

One uses the Householder reflections to compute an orthogonal-triangular factorization: $A * P = Q * R$ where P is a permutation ([8], p.171), Q is an orthogonal matrix, and R is an upper triangular matrix. Let $r := rank(A)$. This algorithm requires $4ML_1r - 2r^2(M + L_1) + 4r^3/3$ flops ([9], pp.248-250). The least-squares solution c is computed by the formula $c = P*(R^{-1}*(Q'*(A^TB)))$. This minimization procedure is based on the matlab code (see [10]).

In [2] and [3] singular value decomposition was used for minimization of $(5'')$. Here we use the matlab minimization code which is based on a factorization of the matrix A. This has the following advantages from the point of view of numerical analysis. We can choose an integer r_1:

$$0 < r_1 \leq r,$$

such that the first r_1 rows and columns of R form a well-conditioned matrix when A is not of full rank, or the rank of A is not known (see [10] and [9] for a further discussion of numerical methods for rank determination).

If we choose $x_j \in D,\ j = 1, ..., J$, then

$$F_J(c) = F_J(c_{01}, ..., c_{0J}, c_{11}, ..., c_{1J}, ..., c_{L1}, ..., c_{LJ})$$

$$= \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{j=1}^{J} a_{i_1i_2} |u_{0i_1i_2} + \sum_{l=0}^{L} c_{lj}\psi_{li_1i_2}|^2 w_{i_1i_2} h_1 h_2.$$

The algorithm for finding the minimum of $F_J(c)$ will be same.

2. Piecewise-smooth boundary:

Suppose

$$S = \bigcup_{n=1}^{N} S_n.$$

Then

$$F(c_0, c_1, ..., c_L) = \sum_{n=1}^{N} ||u_0 + \sum_{l=0}^{L} c_l\psi_l||^2_{L^2(S_n)}$$

$$\forall (x, y, z) \in S_n, \quad r^2 = x^2 + y^2 + z^2, \quad \cos\theta = z/r, \quad \tan\varphi = y/x. \tag{11}$$

3. Non-starshaped case:

Suppose S is a finite union of the surfaces, each of which is starshaped with respect to a point $\vec{r^0_n}$,

$$S = \bigcup_{n=1}^{N} S_n.$$

and the the surfaces S_n are given by the equations in local spherical coordinates:

$$S_n: \quad \vec{r} - \vec{r^0_n} = (r_n(\theta_n, \varphi_n)\cos\varphi_n \sin\theta_n,\ r_n(\theta_n, \varphi_n)\sin\varphi_n \sin\theta_n,\ r_n(\theta_n, \varphi_n)\cos\theta_n),$$

$$n = 1, ...N,$$

where $\vec{r^0_n}$ are constant vectors.

Then

$$F(c_0, c_1, ..., c_L) = \sum_{n=1}^{N} ||u_0 + \sum_{l=0}^{L} c_l\psi_l||^2_{L^2(S_n)}.$$

The weight functions $w_n(\theta, \varphi)$ are the same as in (8) since $\vec{r_n^0}$ are constant vectors.

3. Numerical Results

In this Section, four examples examples are given to demonstrate the convergence rates of the MRC algorithm and the dependence of the numerical errors on the shape of S.

Example 1. The boundary S is the sphere of radius 1 centered at the origin. In this example, the exact coefficients are:

$$c_{lm} = -\frac{4\pi i^l j_l(k)}{h_l(k)}\,\overline{Y_{lm}(\alpha)}$$

Let $k = 1, \quad \alpha = (1, 0, 0)$. We choose $n_1 = 20, \ n_2 = 10$.

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.3219	1.6547	0.2785	0.0368	0.0034	0.0003	0.0000	0.0000
err(c)	0.0303	0.0172	0.0020	0.0004	0.0000	0.0000	0.0000	0.0000

where

$$err(c) = (\sum_{l=0}^{L} |c_l^* - c_l|^2)^{\frac{1}{2}}.$$

When $n_1 = 40, \ n_2 = 20$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.3544	1.6562	0.2820	0.0358	0.0036	0.0003	0.0000	0.0000
err(c)	0.0147	0.0076	0.0011	0.0001	0.0000	0.0000	0.0000	0.0000

Next, we fix $n_1 = 20, \ n_2 = 10$ and test the results for different k and α. When $k = 2, \quad \alpha = (1, 0, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	10.4506	5.5783	1.9291	0.5217	0.0970	0.0156	0.0020	0.0003
err(c)	0.0404	0.0205	0.0048	0.0020	0.0005	0.0000	0.0000	0.0000

When $k = 1, \quad \alpha = (0, 1, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.3801	1.6628	0.2821	0.0371	0.0044	0.0003	0.0000	0.0000
err(c)	0.0014	0.0106	0.0005	0.0004	0.0000	0.0000	0.0000	0.0000

When $k = 1, \quad \alpha = (0, 0, 1)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.4156	1.6909	0.2955	0.0418	0.0025	0.0002	0.0000	0.0000
err(c)	0.0093	0.0109	0.0049	0.0007	0.0001	0.0000	0.0000	0.0000

When $k = 1, \quad \alpha = (1/\sqrt{2}, 1/\sqrt{2}, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.3500	1.6711	0.2810	0.0371	0.0040	0.0003	0.0000	0.0000
err(c)	0.0218	0.0057	0.0019	0.0004	0.0001	0.0000	0.0000	0.0000

When $k = 1, \quad \alpha = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	6.3739	1.6542	0.2850	0.0368	0.0040	0.0003	0.0000	0.0000
err(c)	0.0170	0.0054	0.0021	0.0003	0.0001	0.0000	0.0000	0.0000

Example 2. The boundary S is the surface of the cube $[-1, 1]^3$. Here

$$S = \bigcup_{n=1}^{6} S_n.$$

and

$$F(c_0, c_1, ..., c_L) = \sum_{n=1}^{6} ||u_0 + \sum_{l=0}^{L} c_l \psi_l||^2_{L^2(S_n)}$$

$$= \sum_{n=1}^{6} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2} + \sum_{l=0}^{L} c_l \psi_{l i_1 i_2}|^2 \Delta_1 \Delta_2$$

where

$$\Delta_1 = 2/n_1, \qquad \Delta_2 = 2/n_2.$$

The origin is chosen at the center of symmetry of the cube. The surface area element is calculated in the Cartesian coordinates, so the weight $w = 1$.

Let S_1 be the surface

$$z = 1, \ -1 \le x \le 1, \ -1 \le y \le 1,$$

and

$$x_{i_1} = -1 + i_1\Delta_1, \ 0 \le i_1 \le n_1,$$
$$y_{i_2} = -1 + i_2\Delta_2, \ 0 \le i_2 \le n_2.$$

Then

$$\psi_{li_1i_2} = Y_l(\theta_{i_1}, \varphi_{i_2})h_l(kr(\theta_{i_1}, \varphi_{i_2})),$$

and θ_{i_1} and φ_{i_2} can be computed by formula (11). For other surfaces S_j the algorithm is similar.

The values of $\min F(c) = F(c*)$ and the values of $\min F_J(c) = F_J(c*)$ with the following values of x_j:

$$\{x_j : j = 0, ..., 6\} = \{(0,0,0), (0.2,0,0), (-0.2,0,0),$$
$$(0,0.2,0), (0,-0.2,0), (0,0,0.2), (0,0,-0.2)\},$$

are given below.

We choose $n_1 = 10, \ n_2 = 10$. Then

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
$F(c^*)$	10.6301	3.6277	2.6760	2.2309	1.9832	1.5737	1.5034	1.2948	1.1753
$F_J(c^*)$	2.6297	1.0970	0.5487	0.1572	0.0667	0.0320	0.0168	0.0078	0.0035

When $n_1 = 20, \ n_2 = 20$, then

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
$F(c^*)$	10.7923	3.7144	2.7778	2.3393	2.0873	1.6671	1.5938	1.4277	1.3368
$F_J(c^*)$	2.7248	1.1433	0.5757	0.1686	0.0694	0.0652	0.0236	0.0143	0.0090

Example 3. The boundary S is the surface of the ellipsoid $x^2+y^2+z^2/b^2 = 1$, the values of $\min F(c) = F(c*), \ b = 2,3,4,5$ with $n_1 = 20, \ n_2 = 10$ are:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
b=2	8.8836	5.4955	3.0421	2.8434	1.3622	1.2093	0.8753	0.8132

b=3	14.1617	12.0477	7.2296	7.0999	3.8077	3.6829	3.1324	3.0496
b=4	19.5326	17.9346	9.9927	9.8720	5.3333	5.2008	4.6793	4.5738
b=5	22.9765	21.5653	11.4850	11.3587	6.1637	6.0096	5.5202	5.3933

The values of $\min F_J(c) = F_J(c*)$, $b = 2, 3, 4, 5$ with the following values of x_j:

$$\{x_j : j = 0, ..., 6\} = \{(0,0,0), (0.5,0,0), (-0.5,0,0),$$
$$(0,0.5,0), (0,-0.5,0), (0,0,0.5), (0,0,-0.5)\},$$

are:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
b=2	2.4856	0.7090	0.2530	0.0062	0.0000	0.0000	0.0000	0.0000
b=3	4.6639	1.3619	0.6618	0.0074	0.0000	0.0000	0.0000	0.0000
b=4	5.5183	1.8624	0.7844	0.0060	0.0000	0.0000	0.0000	0.0000
b=5	11.0579	8.7027	6.4831	0.8357	0.0017	0.0000	0.0000	0.0000

Example 4. The obstacle is a *dumbbell*. Its boundary S is *not smooth, not star-shaped and not convex*:

$$S = S_1 \bigcup S_2 \bigcup S_3$$

$$S_1 : \ \vec{r} - (0,0,1) = (1.5\cos\varphi\sin\theta, 1.5\sin\varphi\sin\theta, 1.5\cos\theta)$$
$$S_2 : \ \vec{r} - (0,0,-1) = (1.5\cos\varphi\sin\theta, 1.5\sin\varphi\sin\theta, 1.5\cos\theta)$$
$$S_3 : r\sin\theta = 1$$
$$\{x_j : j = 0, ..., 10\} = \{(0,0,0), (0,0,0.1), (0,0,-0.1), (0,0,0.2), (0,0,-0.2),$$
$$(0,0,0.3), (0,0,-0.3), (0,0,0.4), (0,0,-0.4), (0,0,0.5), (0,0,-0.5)\};$$

We choose $n_1 = 20$, $n_2 = 10$ for every $S_i, (i = 1, 2, 3)$. Then:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
$F(c^*)$	25.8840	20.8059	16.4968	15.6622	12.9241	12.1915	11.0187	9.5263
$F_J(c^*)$	20.3118	8.0238	5.1062	2.5908	0.8304	0.4067	0.0453	0.0084

4. Conclusion

From the numerical results one can see that the accuracy of the numerical solution depends on the smoothness and elongation of the object.

In Example 1 the surface S is a unit sphere and the numerical solution is very accurate. In Example 3 the results for different elongated ellipsoids show that if the elongation (eccentricity) grows, then the accuracy decreases. In Example 2 the surface is not smooth and the result is less accurate than in Example 3. In Example 4 the surface is not convex and not smooth, but the accuracy is of the same order as in Example 2.

When b is large or S is not smooth, the numerical results in Example 2 and Example 3 show that if one adds more points x_j, then the accuracy of the solution increases.

In Examples 1 and 2, as n_1 and n_2 grow, the minimum $F(c*)$ grows because the condition number of the matrix A in (10) grows as n_1 and n_2 grow.

Using the results of Example 1 one can check the accuracy in finding c_l by the value of the minimum

$$F(c*) \leq \epsilon.$$

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